DETERMINATION OF THE STATISTICAL CHARACTERISTICS OF THE CHAOTIC MOTION OF PARTICLES IN A FLUIDIZED BED

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A method of automation of the measurement of the effective diffusion coefficient in a fluidized bed is proposed and developed.

In our previous studies [1, 2] we made a quantitative division of the two main components of the motion of particles in a fluidized bed — the circulation motion of groups of neighboring particles through the entire apparatus and chaotic motion of the diffusion type caused by the random nature of the forces of interparticle interaction through the stream and leading to the decay and regeneration of "packets" of particles. The statistical nature of the processes leads to the fact that it is necessary to analyze a large number of measurements of the successive positions of a marked particle in the reactor and the distances between these positions in order to determine the mean values of the parameters of these motions — the circulation velocity v and the effective particle diffusion coefficient D. Other groups of investigators studying various statistical characteristics of the motion of particles in a fluidized bed [3-5] have encountered similar difficulties. Regardless of the method of obtaining the initial data (by direct measurements or the analysis of movie frames) the data for the statistical analysis must be obtained "by hand" and only then transferred to punched cards for subsequent computer processing.

The great laboriousness of such methods has raised the problem of the automation of the measurements, recording, and computation of the initial data. With some change in the formulation of the experiment this course turns out to be feasible and promising.

By definition, the effective diffusion coefficient is obtained through the averaging of the ratio $D = \alpha(l^2/t)$. When filming with equal intervals t between frames it is necessary to measure by hand all the successive segments l_i and average over them, i.e., to assume that

$$D = \frac{\alpha}{t}\,\overline{l^2} = \frac{\alpha}{t}\,\frac{1}{N}\sum_{i=1}^{N} l_i^2. \tag{1}$$

One can, however, formulate the experiment differently and make the particle close some time relay at the moment it reaches a given point of the reactor and then open this relay when the particle travels a fully defined distance l (i.e., crosses a sphere or circle of radius l). The successive intervals t_i obtained can be recorded automatically and the unknown diffusion coefficient determined through the averaging

$$D = \alpha^* l^2\,\overline{\left(\frac{1}{t}\right)} = \alpha^* l^2\,\frac{1}{N}\sum_{i=1}^{N}\frac{1}{t_i}. \tag{2}$$

To find the numerical factor α^* in this case one must solve the diffusion problem

$$\frac{\partial c}{\partial t} = D\nabla^2 c\ , \tag{3}$$

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with the initial condition that the particle lies at the origin of coordinates and the probability density of finding it is described by the Dirac δ-function,

$$c(r,\ 0)=\delta(r). \tag{4}$$

When the particle reaches the boundary L of the region the circuit of the time relay opens and the particle is absorbed by the contour, as it were, which corresponds to the boundary condition

$$c_L(t)=0. \tag{5}$$

By denoting the distance to the boundary (the radius of the region) as l and changing to the dimensionless variables

$$\vec{\xi}=\frac{\vec{r}}{l};\quad \tau=\frac{Dt}{l^2}, \tag{6}$$

one can write the solution of Eq. (3) with the conditions (4) and (5) in the form

$$c=W(\xi,\ \tau). \tag{7}$$

The total diffusion flux passing through the entire contour boundary,

$$\oint_L jld\eta dt=-\oint D\left.\frac{\partial c}{\partial n}\right|_{r=l} ld\eta dt\sim\oint_L \left.\frac{\partial W(\xi,\ \tau)}{\partial \nu}\right|_{\xi=1} d\eta d\tau=\Psi(\tau)\,d\tau, \tag{8}$$

gives the probability that the particle travels a given length l from the center to the boundary of the region in the time interval from τ to $\tau + d\tau$. Having the probability density function $\Psi(\tau)$, one can calculate not only the mean value of the inverse time

$$\overline{\left(\frac{1}{t}\right)}=\frac{D}{l^2}\cdot\frac{\int\limits_0^\infty \frac{1}{\tau}\Psi(\tau)\,d\tau}{\int\limits_0^\infty \Psi(\tau)\,d\tau}=\frac{D}{l^2}\cdot\frac{1}{\alpha^*}, \tag{9}$$

but also any other mean characteristics $\bar{t}$, $\bar{t}^2$, or the most probable times of motion $t_p = (l^2/D)\tau_0$, where τ_0 is determined from the equation

$$\frac{\partial\Psi(\tau)}{\partial\tau}=0. \tag{10}$$

Since the function $\Psi_{exp}(\tau)$, obtained experimentally on the basis of a large but finite number of measurements, will not coincide exactly with the calculated dependence (8), in a comparison of the experimentally found relationships between $\overline{(1/t)}$, $\bar{t}$, $\bar{t}^2$, and t_p with the analogous relationships between the calculated values of $\overline{(1/\tau)}$, $\bar{\tau}$, $\bar{\tau}^2$, and τ_0 one will observe a certain scatter of the values of the effective diffusion coefficient D calculated in this way, depending on the accuracy of the measurements.

In the presence of circulation motions the problem becomes more complicated, since Eq. (3) must be replaced by the more general equation

$$\frac{\partial c}{\partial t}=D\nabla^2 c-\vec{v}\nabla c. \tag{11}$$

With the change to dimensionless variables the problem and its solution prove to be dependent on the additional dimensionless Péclet number

$$\mathrm{P}=\frac{vl}{D}. \tag{12}$$

The probability density of the time of travelling the distance l, i.e., the function $\Psi(\tau,\ \mathrm{P})$, and consequently all the characteristics $\overline{\tau^n}$ and τ_0, will also depend on this quantity. Therefore, in a comparison between experimentally averaged times and calculated times it is necessary to calculate at least two different characteristics, and to determine the quantity D one must construct a combination of them from which the quantity P has been eliminated on the basis of a theoretical analysis.

Theoretical Statement of the Problem

Such a problem was first solved completely by Smolukhovskii. In [6] he considered a heavy particle which enters the gravitational field with constant velocity from a point located at a height l above the ground level and undergoes Brownian motion characterized by the effective diffusion coefficient D. By solving Eq. (11) with the added conditions (4) and (5) for this case Smolukhovskii found the probability distribution for the times in which the incident particle reaches the earth's surface. In the dimensionless variables (6) presented above this function (8) has the form

$$\Psi(\tau)\,d\tau=\frac{1}{2\sqrt{\pi\tau^3}}\exp\left[-(1-P\tau)^2(4\tau)^{-1}\right]d\tau. \tag{13}$$

Using this function, one can easily determine the unknown time characteristics

$$\begin{gathered}\bar{t}=\frac{l}{v};\quad \overline{\left(\frac{1}{t}\right)}=\frac{v}{l}+\frac{2D}{l^2};\quad \overline{t^2}=\frac{l^2}{v^2}+\frac{2Dl}{v^3};\\ t_p=\frac{3D}{v^2}\left[\sqrt{1+\frac{P^2}{9}}-1\right].\end{gathered} \tag{14}$$

The values of v and D can be determined in various ways from these equations.

Thus, $v = l/\bar{t}$, and on the suggestion of Weiss [7], we have

$$D=\frac{l^2}{2}\left[\frac{1}{\bar{t}}-\overline{\left(\frac{1}{t}\right)}\right]. \tag{15}$$

For $P \lesssim 1$ it is convenient to determine D approximately from (14) with an accuracy of 3% by the relation

$$D\approx\frac{l^2}{6t_p}. \tag{16}$$

As seen from this, for $P < 1$ the relation (16) is the least sensitive to the effect of the velocity of the ordered motion for a determination of D. In the limiting case of $P \to \infty$, when $v \gg D/l$, the distribution (13) approaches the δ function and all the average values are identical:

$$[\overline{(t^{-1})}]^{-1}=\bar{t}=(\overline{t^2})^{1/2}=t_p=\frac{l}{v}.$$

In the other limiting case of $P \to 0$, when $v \ll D/l$,

$$[\overline{(t^{-1})}]^{-1}=\frac{l^2}{2D},\quad t_p=\frac{l^2}{6D},$$

and all the residual averages $\overline{t^n}$ for $n > 0$ approach infinity, so that the "tail" of the distribution (13) is strongly extended in this case, declining only as $t^{-3/2}$, and the diffusing particles can at first depart very much from the boundary, until they randomly turn around and approach it.

The limiting region of wandering of the marked particle is of great interest for our problems. In the one-dimensional case in the dimensionless variables (6) with the parameter (12), Eq. (11) takes the form

$$\frac{\partial c}{\partial\tau}=\frac{\partial^2 c}{\partial\xi^2}-P\frac{\partial c}{\partial\xi} \tag{17}$$

with the initial condition (4) and the following boundary conditions on both sides of the interval:

$$c(\pm 1,\,\tau)=0\quad\text{at}\quad \xi=\pm 1. \tag{18}$$

Using the operator method we introduce the following transform of the unknown function:

$$Z(s,\,\xi)=\int_0^\infty\exp(-s\tau)\,c(\xi,\,\tau)\,d\tau. \tag{19}$$

Substituting into (17) and using the initial and boundary conditions, we find that

$$Z(s,\ \xi)=\frac{\exp\left(\frac{P}{2}\xi-\sqrt{s+\frac{P^2}{4}}\right)}{2\sqrt{s+\frac{P^2}{4}}}\left[\exp\left(1-|\xi|\sqrt{s+\frac{P^2}{4}}\right)-\frac{\operatorname{ch}\xi\sqrt{s+\frac{P^2}{4}}}{\operatorname{ch}\sqrt{s+\frac{P^2}{4}}}\right]. \quad (20)$$

The total flux through both boundaries is

$$\Psi(t)=-D\frac{\partial c}{\partial x}\bigg|_{x=+l}+D\frac{\partial c}{\partial x}\bigg|_{x=-l}$$

and its transform in the dimensionless coordinates (6) is

$$\Phi(s)=\frac{dZ}{d\xi}\bigg|_{\xi=-1}-\frac{dZ}{d\xi}\bigg|_{\xi=+1}=\frac{\operatorname{ch}\frac{P}{2}}{\operatorname{ch}\sqrt{s+\frac{P^2}{4}}}. \quad (21)$$

The inverse transform is represented in the form of the infinite series

$$\Psi(\tau)=\operatorname{ch}\frac{P}{2}\exp\left(-\frac{P^2}{4}\tau\right)\pi\sum_{0}^{\infty}(-1)^k(2k+1)\exp\left[-\frac{\pi^2}{4}(2k+1)^2\tau\right]. \quad (22)$$

Since this series converges very rapidly even when $\tau > 0.1$, one can seek the time of reaching the maximum probability of the arrival of the particle at the boundary by differentiating only the first two terms of the series and equating the derivative of this sum to zero. From this condition it is easy to find the quantity

$$\tau_0=\frac{1}{2\pi^2}\ln\left[27\frac{1+\frac{P^2}{9\pi^2}}{1+\frac{P^2}{\pi^2}}\right]=0.1467+0.0507\ln\frac{1+\frac{P^2}{9\pi^2}}{1+\frac{P^2}{\pi^2}}, \quad (23)$$

which when $P < 1$ equals 0.147 with an accuracy of 3%, i.e., close to $^1/_6 = 0.167$ as in (16).

The series (22) obtained is not suitable for finding the mean values of the times $\overline{\tau^n}$, and it is convenient to start directly from its transform (21).

From the definition of the average

$$\overline{\tau^n}=\frac{\int_0^\infty\tau^n\Psi(\tau)\,d\tau}{\int_0^\infty\Psi(\tau)\,d\tau}.$$

First let us find $\int_0^\infty\Psi(\tau)\,d\tau$. If the transform of the function $\Psi(\tau)$ is $\Phi(s)$, then the transform of the unknown integral will be $(1/s)\Phi(s)$. As is known, large values of τ correspond to small values of the Laplace parameter s. Consequently, the transform of the number $\int_0^\infty\Psi(\tau)\,d\tau$ of interest to us will be the limit $\frac{1}{s}\lim_{s\to 0}\Phi(s)=\frac{\Phi(0)}{s}$, while the number itself will be

$$\int_0^\infty\Psi(\tau)\,d\tau=\Phi(0). \quad (24)$$

But in our case $\Phi(0) = ch(P/2)/ch(P/2) = 1$.

Similarly, one can find that when $n > 0$

$$\overline{\tau^n} = \int_0^\infty \tau^n \Psi(\tau)\, d\tau = (-1)^n \left.\frac{d^n \Phi(s)}{ds^n}\right|_{s=0},$$

$$(\overline{\tau^{-1}}) = \overline{\left(\frac{1}{\tau}\right)} = \int_0^\infty \Phi(s)\, ds. \tag{25}$$

Thanks to the boundedness of the region on both sides $-l < x < +l$ in this example, in contrast to the preceding Smolukhovskii problem, the average times $\overline{t^n}$ (for $n > 0$) do not go to infinity when $v = 0$. On the other hand, in comparison with (14) the definite additivity of the diffusion and convective components disappears from the expressions for these times. In fact, in this case

$$\tau = -\left.\frac{d\Phi}{ds}\right|_{s=0} = \frac{1}{2\sqrt{s+\frac{P^2}{4}}} \cdot \frac{\operatorname{ch}\frac{P}{2}\sqrt{s+\frac{P^2}{4}}}{\operatorname{ch}^2\sqrt{s+\frac{P^2}{4}}} = \frac{1}{2}\cdot\frac{\operatorname{th}\frac{P}{2}}{\frac{P}{2}} \begin{cases} \to \frac{1}{2} & \text{as } P\to 0, \\ \to \frac{1}{P} & \text{as } P\to\infty. \end{cases}$$

Changing to dimensional variables, we obtain

$$\bar{t} = \frac{l^2}{2D}\cdot\frac{\operatorname{th}\frac{vl}{2D}}{\frac{vl}{2D}} \begin{cases} \to \frac{l^2}{2D} & \text{when } v \ll \frac{D}{l}, \\ \to \frac{l}{v} & \text{when } v \gg \frac{D}{l}. \end{cases}$$

Similarly, for the average inverse time

$$(\overline{\tau^{-1}}) = \operatorname{ch}\frac{P}{2}\int_0^\infty \frac{ds}{\operatorname{ch}\sqrt{s+\frac{P^2}{4}}} = \operatorname{ch}\frac{P}{2}\int_{P/2}^\infty \frac{2y\,dy}{\operatorname{ch} y} \begin{cases} \to 3{,}64 & \text{as } P\to 0. \\ \to 2+P & \text{as } P\to\infty. \end{cases}$$

The complicated dependences of $\bar{t}$ and $(\overline{t^{-1}})$ on D and v which are obtained make difficult the separate determination of the latter parameters from the experimental data for the average times. Thus, if one multiplies these times, one obtains the relation

$$\bar{t}\,(\overline{t^{-1}}) = \bar{\tau}\,(\overline{\tau^{-1}}) = \frac{\operatorname{sh}\frac{P}{2}}{P}\int_{P/2}^\infty \frac{2y\,dy}{\operatorname{ch} y} = F(P), \tag{26}$$

from which, in principle, one can determine the quantity P, i.e., the ratio vl/D, although the function F(P) which stands on the right and which can be tabulated varies relatively slowly in the entire range of variation of the argument (from 1.82 as $P \to 0$ to 1 as $P \to \infty$) and small errors in the experimental determination of the quantities $\bar{t}$ and $(\overline{t^{-1}})$ and their product lead to an error of hundreds of percent in the determination of P.

It is natural that in this case also as $P \to \infty$ ($v \gg D/l$) the probability distribution $\Psi(t)$ of the times in which the marked particle reaches the boundary of the region approaches the δ function and all the time averages,

$$\overline{t^n} \to \left(\frac{l}{v}\right)^n.$$

Thus, in the presence of outer absorbing boundaries of the region, if the experimental distribution function is relatively broad and far from the peak-shaped δ function, then one can be sure that the Péclet number is small ($P \lesssim 1$) and one can determine the effective diffusion coefficient D from the experimental data by assuming that $P = 0$. The most accurate and convenient method in this case is the use of Eq. (23), from which we have

$$D_e = 0.147 \frac{l^2}{t_p}. \quad (27)$$

To estimate the degree of influence of the convective component (the quantity P) on the accuracy of the value of D_e obtained one can compare how the quantities $D_e t/l^2$ and $(l^2/D_e)(\overline{1}/t)$ differ from their limiting values $^1/_2$ and 3.64 and how the product $t(t^{-1})$ differs from 1.82.

For the determination of the convective characteristic v it is necessary to increase the Péclet number $P = vl/D$ as much as possible under the experimental conditions, i.e., to move out the boundary l of the measurement region.

The equations and the conclusions obtained in the last example will also be qualitatively valid in the two- and three-dimensional cases with cylindrical and spherical boundaries. These considerations served as the basis for the development of an automated method of measuring the effective diffusion coefficient for particles in a fluidized bed.

Automation of Statistical Measurements of Transit Times

The experiments were performed, as earlier, on a model installation containing a two-dimensional fluidized bed "one grain thick." Initially we used the method of filming a particle marked with dye and moving among 100 other aluminum disks just like it, 10 mm in diameter and 5 mm thick, in a plastic reactor 500 mm high and with a rectangular cross section of 150 × 7 mm. The two-dimensional bed was fluidized by an ascending stream of water or aqueous solutions of glycerin, circulated through the closed system. The motion-picture frames were analyzed by hand and the successive movements l_i of the marked particle were measured at different fixed intervals t (every frame, every two frames, every three frames, etc.). The values of the parameters D and v were determined by a method described earlier [8]. With stream velocities on the order of 10 cm/sec it was found that $D \approx 5$ cm²/sec and $v \approx 7$ cm/sec.

We chose an optical method for the automation of the measurements of t. The main particles of the bed were transparent isotropic glass disks 8 mm in diameter. The marked particle was a disk of the same mass and size made of quartz, which rotates the plane of polarization. The thickness of this disk (and, consequently, of the other glass disks) had to be 3.8 mm for it to rotate light with a wavelength of 550-560 nm by 90°. The model transparent reactor had a height of 480 mm and a rectangular cross section of 150 × 6 mm. The bed of 200 polished glass disks and one quartz disk poured into it was fluidized by an ascending stream of water or a mixture of water and glycerin.

A schematic optical diagram of the installation is shown in Fig. 1b. Light from the incandescent lamp 1 passes through the light filters 2-2', which isolate the section of the spectrum indicated above, the long-focus lens 3, the polarizer 4, and the opaque screen 5 with an opening 11 at the center and an annular slot 10 (see Fig. 1a), and it illuminates the reactor 6. The analyzer 7 is crossed with the polarizer and almost extinguishes the light passing through the reactor and the isotropic glass beads. Only when the quartz disk is in the path of the beam is its plane of polarization rotated so that the beam passes through the analyzer.

When the quartz particle crosses over the central opening of the screen which is 8 mm in diameter the emerging light falls on the photoresistor 8, the resistance of the latter drops sharply, and a voltage pulse arises in its circuit which enters a special electronic selector (a diagram of which is not presented here), turning on a block of nine electromechanical scalers. Each of these scalers waits for its set time t_k and then turns off, turning on the next scaler.

After some time t the quartz particle, having traveled a length l, crosses a section of the annular slot, the large diameter of which is 55 mm and the smaller diameter 45 mm. The light from these sections is focused by the second lens 3 on a second photoresistor 9, which sends a second pulse to the electronic circuit. If this time $t > \sum_{i=1}^{k} t_i$, but $t < \sum_{i=1}^{k+1} t_i$, then the first k scalers have already turned off, the (k + 1)-th waiting scaler receives the pulse and its pointer shifts by one division, and the remaining counters have not yet been turned on. This same pulse switches the system to the initial state and the circuit is ready for a new triggering, when the quartz particle again crosses the central opening of the screen.

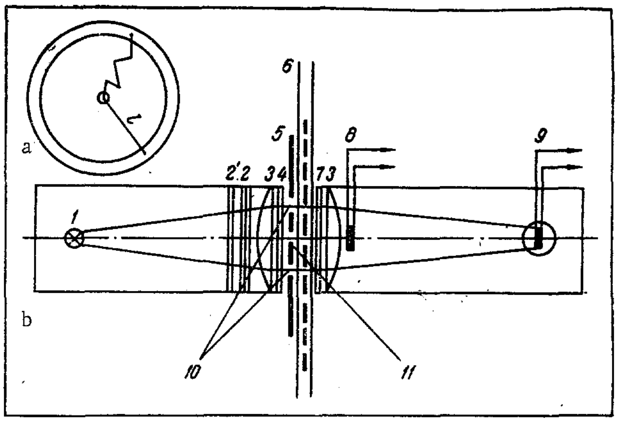

Fig. 1. Trajectory of motion of marked particle from "center" to "ring" (a) and optical diagram of experimental installation (b).

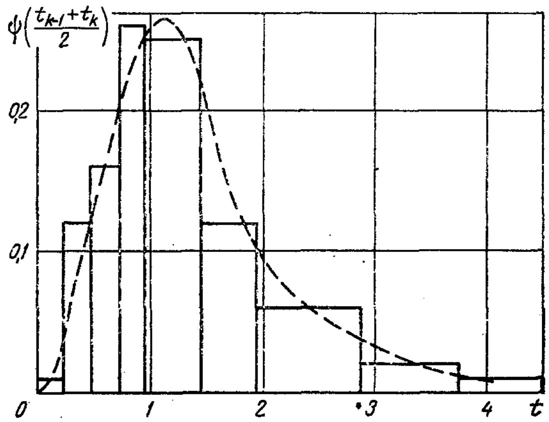

Fig. 2. Histogram of the times in which the marked particle passes a given distance l (the fluidizing substance is a 70% solution of glycerin in water). t, sec.

At the end of the experiment, which lasts 2-3 h, the readings n_k of all the scalers, representing the numbers of cases when the marked particle traveled a given length l in a time t in the interval $\Delta t_k = t_k - t_{k-1}$, are taken and recorded. The quantity $n_k/\Sigma n_i$ itself characterizes the value $\Psi(t)dt$, introduced above, of the probability distribution of the times in which the particle travels a given path length (or more accurately, displacement) l. The duration of each experiment was chosen so that the time selector on the nine scalars recorded 100-150 times of passage of the marked particle from the center to the ring in this time. For convenience of the count the intervals Δt_k between the times of switching of the scalers were chosen as different and increasing for larger t_k. The values of the distribution function corresponding to this were determined as

$$\Psi\left(\frac{t_{k-1}+t_k}{2}\right)=\frac{n_k}{\Delta t_k\sum\limits_1^9 n_k} \tag{28}$$

and the average values were calculated from the equations

$$\overline{(t^n)}=\frac{\sum\limits_1^9\left(\frac{t_{k-1}+t_k}{2}\right)^n n_k/\Delta t_k}{\sum\limits_1^9 n_k/\Delta t_k}. \tag{29}$$

A histogram of one of these experiments is presented in Fig. 2.

TABLE 1. Results of Measurements of the Statistical Characteristics of the Chaotic Motion of Particles in a Fluidized Bed

Composition of fluidizing material	Viscosity × 10^6, m^2/sec	Archimedes number	Flow rate × 10^3, m^3/sec	$\overline{\left(\frac{1}{t}\right)}$, sec^{-1}	$\bar{t}$, sec	$\sqrt{\overline{t^2}}$, sec	t_p, sec	$D=0{,}163 l^2 \overline{\left(\frac{1}{t}\right)} \times 10^4$, m^2/sec	$D=0{,}25 \frac{l^2}{\bar{t}} \times 10^4$, m^2/sec	$D=0{,}3 \frac{l^2}{\sqrt{\overline{t^2}}} \times 10^4$, m^2/sec	$D=0{,}12 \frac{l^2}{t_p} \times 10^4$, m^2/sec	$\bar{t}\overline{\left(\frac{1}{t}\right)}$	$\bar{D}\times 10^4$, m^2/sec	$\pm\Delta\bar{D}\times 10^4$, m^2/sec	Number of measurements
Water	1	$5{,}8\cdot 10^6$	0,10	3,12	0,75	0,80	0,36	3,16	2,08	2,32	2,08	2,34	2,8	0,6	103
45% solution of glycerin in water	5	$5\cdot 10^5$	0,14	0,92	1,44	1,62	1,2	0,94	1,08	1,15	0,63	1,31	1,0	0,2	102
Same	5	$5\cdot 10^5$	0,19	1,79	0,86	1,04	0,36	1,82	1,81	1,78	2,08	1,54	1,9	0,1	105
"	5	$5\cdot 10^5$	0,22	2,38	0,66	0,85	0,36	2,39	2,36	2,19	2,08	1,56	2,2	0,1	150
70% solution of glycerin in water	16	$5\cdot 10^4$	0,11	0,96	1,24	1,38	1,0	1,56	1,26	1,35	0,75	1,19	1,2	0,2	100
Same	16	$5\cdot 10^4$	0,12	1,06	1,35	1,61	0,7	1,08	1,15	1,15	1,07	1,43	1,11	0,04	160
90% solution of glycerin in water	16	$5\cdot 10^4$	0,13	1,30	1,12	1,31	0,8	1,33	1,39	1,42	0,94	1,45	1,3	0,2	103
Same	50	$5\cdot 10^3$	0,04	0,59	2,42	2,64	1,5	0,60	0,64	0,70	0,50	1,42	0,61	0,06	100
"	50	$5\cdot 10^3$	0,06	0,83	2,04	2,36	1,2	0,85	0,76	0,79	0,63	1,70	0,76	0,06	100

Results of Measurements

For the determination of the effective diffusion (dispersion) coefficient for the chaotic motion of particles it would be desirable to perform the experiments with small values of the Péclet number in order to eliminate or sufficiently weaken the influence of the circulation motion of "packets" of particles through the reactor. For this we chose the radius l = 2.5 cm for the ring. From an estimate of the values of D and v obtained in the experiments on the motion of aluminum disks in similar liquids [8], we have

$$P=\frac{vl}{D}\approx\frac{7\cdot 2.5}{5}\approx 3-4.$$

To obtain the calculating equations in our case of cylindrical symmetry we solved the diffusion equation corresponding to the limiting case of P = 0,

$$\frac{\partial c}{\partial t}=D\left[\frac{\partial^2 c}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial c}{\partial r}\right] \quad (30)$$

with the initial condition $c(r, 0) = \delta(r)$ and the boundary condition $c(l, t) = 0$.

The solution was carried out by the operator method and for the transform of the total flux $\Psi(t)$ through the entire annular boundary of the region we obtained

$$\Phi(s)=\frac{1}{I_0\left(\sqrt{\frac{s}{D}}\,l\right)}, \quad \Phi(0)=1. \quad (31)$$

The inverse of this transform can be represented in the form of the rapidly converging series

$$\Psi(t)=-\frac{2D}{l}\sum_{1}^{\infty}\frac{\gamma_k}{J_1(\gamma_k)}\exp\left(-\gamma_k^2\frac{Dt}{l^2}\right), \quad (32)$$

where γ_k are the roots of the zeroth-order Bessel function $J_0(\gamma_k) = 0$. Confining oneself to the first two terms of this series, one could find the time in which the maximum value of the probability $\Psi(t)$ is reached:

$$t_p\approx 0.12\frac{l^2}{D}. \quad (33)$$

The average times were calculated from Eqs. (25) using the transform (31). The integral of (25) was calculated numerically. As a result, we obtained

$$\overline{\left(\frac{1}{t}\right)}=6.13\frac{D}{l^2};\quad \bar{t}=0.25\frac{l^2}{D};$$

$$\sqrt{\overline{(t^2)}}=0.31\frac{l^2}{D};\quad \bar{t}\,\overline{\left(\frac{1}{t}\right)}=1.53. \quad (34)$$

From this the quantity D could be calculated by four different means:

$$D = 0.163 l^2 \overline{\left(\frac{1}{t}\right)}; \quad D = 0.25\,\frac{l^2}{\bar{t}};$$
$$D = 0.31\,\frac{l^2}{\sqrt{\overline{(t^2)}}}; \quad D = 0.12\,\frac{l^2}{t_p}. \tag{35}$$

The results of these calculations for different fluidization conditions are summarized in Table 1.

As in the example of the one-dimensional linear problem analyzed above, in the case of cylindrical symmetry the product $\bar{t}(\overline{1/t})$ should be a function of the Péclet number and should decrease from the calculated value of 1.53 at P = 0 to 1 as $P \to \infty$. The intermediate values of this product of 1.1-1.5 obtained in the experiment show that the value of P is actually on the order of unity, but they do not permit one to accurately determine its value and thereby the values of the circulation velocity $v = P(D/l)$. In addition, by the very method of the experiment a considerable time interval of about 100 sec passes between individual measurements of t until the marked particle again reaches the initial point of origin of reckoning of the travel. But the quantity v in a fluidized bed is only an average characteristic of the circulation flows of the solid phase and in the interval between counts it can change markedly both in magnitude and in direction.

NOTATION

v, circulation velocity; D, effective diffusion coefficient; l, travel of a particle in a time t and mean radius of the annular slot; t, t_p, time interval and its probable value; α, α^*, numerical coefficients which depend on the statement of the problem; c, concentration of the marked admixture; n, normal to the contour; r, cylindrical coordinate; L, size of region; ξ, τ, ν, $d\eta$, dimensionless coordinate, dimensionless time, dimensionless normal, and dimensionless element of the contour; j, diffusion flux; $\Psi(\tau)$, probability density function; P, Péclet number; I_0, J_0, J_1, Bessel function of imaginary and real argument; γ_k, roots of Bessel function; n_k, readings of k-th scaler.

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